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Machine Learning and Student's Educational Trajectory Mathematical Modelling

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Abstract
The purpose of the article is to describe stages of variables formation and identification and develop a student's educational trajectory mathematical model and to create a technological model for the application of machine learning methods to predict the optimal educational trajectory of the student. External variables are analyzed, variables' numerical values are determined, types of collecting information sessions are highlighted. Data arrays from information sources electronic diary Diary.ru and the GlobalLab online platform were analyzed. A mathematical model of the student is presented based on variables. Using the technical and scientific results of the Project will provide useful economic, technological and technical effects.
Keywords: machine learning, mathematical model, GlobalLab, online training, educational trajectory

I. INTRODUCTION
The educational process' methods, forms and structure are usually designed for the average student, however, the time spent on the individual assimilation of certain educational information may not be the same as official schedule' average time allotted on it. Individualization of education is achieved by machine learning (ML) [12]. Methods and algorithms of ML are successfully and widely used for:
- preparation of students for real-life situations [2];
- reducing the cost of individual training;
- decreasing bias in grading;
- identification what students need assistance and what teachers are available;
- giving a feedback to teachers and students [13];
- formation of personal educational paths' through automation;
- increasing positive educational results;
- incorporating technology into teaching with a minimum of equipment;
- support and improvement of decision-making and government officials on national level [27];
- computer vision, robotics, social network analysis [28];
- development of hardware and software skills;
- development of design and planning skills;
- spending less time on learning.

Though, small budget independent teams and researchers can`t get access to training data in large quantities. The decision is using leverage weak annotations, of lower quality, that are easier to collect for ML through several Weakly Supervised Learning (WSL) techniques for image-based data.
- Spending less effort and money to gain professional skills.
- Technology can be beneficial for the students' mental health: for instance, conducting virtual tours to the space of secondary school has shown promising results for reducing the children's anxiety in the transition from primary to secondary school [22].
- Creating an educational environment rich in information-communicative technologies is the key factor for the development of the information society in any country [23].
- The information-communicative technologies themselves are the means for moderating the effects on innovative entrepreneurship on progress and, consequently, improving the quality of life and stimulating human development [24].

ML has an ability of learning how individuals' features, including social information, can be mapped into personalized predictions of risks [27].

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Campus units provide different types support, integrated into assistant development programs and existing teachers. The department or the faculty support research or media education when it needs the performance of function of a service unit [13].
ML support has such forms:
- desk or traditional support;
- short-term training and counseling;
- long-term training and counseling;
- posting training programs on the website of a course;
- displaying slides in Power Point;
- design and production support for development of a basic website;
- technical equipment support;
- classroom equipment support;
- learning management systems;
- strong design support.

So, there are evidences that teaching methods are changing to fit emerging and already existing technologies. a Teachers create three-dimensional interactive Java programs for scientific demonstration. Also, creation of nanomaterials has already started as a task for computer modeling [21].

II. LITERATURE REVIEW

In the 1990s appeared the first works applying ML as mathematical optimization algorithms substitute, this investigation was active until results on combinatorial optimization study [25]. Such scholars as Bibby, Hasper, Luzzo, Martinelli, Albert investigated the effect of experiences in mathematical learning and the performance accomplishments on participants' vocational interests' assessments, their career choice through pre- and post-treatment, applying counseling and theoretical implications, career aspirations. Barber, Eccles, and Jozefowicz investigated the Model of achievement-related choices [2].

Surveys made by PISA annually show real-life context to measure students' action and mathematical thought, fundamental mathematical capabilities: argument and reasoning, mathematisation, devising strategies, communication, representation, using mathematical tools, using symbolic, technical and formal operations and language [4–7]. Stuart Dreyfus elaborated and defended acquisition of skills implying attempts to capture understanding in the form of mathematical models not distorting and degrading the true wisdom of experience [8].

A long history has overcome the concept of a learning trajectory in the developmental psychology. In began with the statement that children are not incomplete adults, not miniature, and understand the world through interactions with others and experiences, views evolve from naive to sophisticated. This recognition led number of scholars to investigate mechanisms of learning. Piaget with his colleagues made a research program documenting the ideas of children. The learning trajectories' development was positioned within socio-constructivist and constructivist learning views [10]. Then Freudenthal has found a Realistic Mathematics Education (RME) scholarly tradition: "mathematical structures are not a fixed datum, they emerge from reality and expand continuously in collective and individual learning processes" [26].

III. METHODOLOGY

GlobalLab online platform arrays of data, the electronic diary Diary.ru were used for the formation of the student's educational trajectory model.
Mathematical modeling methodology, selection and analysis of variables was used. Sessions' types were formed for collecting information on students. Using the k-means method variables' values averaged over all sessions were clustered for every session type.
Jupyter Notebook, GNU Octave software was used for mathematical modeling.

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If models by ML can configure each participant profile, recommendations can be given for creation of the most efficient teams and hence produce the best results.
In this study the major task is to explore the developing online math courses possibility in line with the ideas presented in the introduction.
Literature used is divided into the such groups:
- identifying learning technique research;
- personalized training practical methods on online courses research;
- mental health specific features and processes individualization research;
- humanities and social online education research;
- creating and delivering online courses technical aspects research [11, 12].

IV. RESULT AND DISCUSSION

Stage 1 includes applied scientific research (ASR), all the variables used in mathematical modeling of both the subject and the object of the educational path were proposed to be divided into two main clusters:
1).external variables' cluster;
2).internal variables' cluster.
External variables include variables describing the student's properties, independent of the Stage within the educational path. These properties include learning style, gender, student age. External variables and methods for their values' calculating are a mathematical model of a student as an educational trajectory' subject.

Stage 2 of the ASR is a part of the solution to the problem of developing a mathematical model of a student. At this stage researchers need to:
1). Determine the possibility and method of modeling or restoring the values of a hypothetical external variable if its value cannot be obtained directly from data sources.
2). Consider the hypothetical external variables list. Its values are obtained from data sources available to researchers (GlobalLab platform data, electronic diary Diary.ru data).
3). Determine the number of students for whom data are available for each hypothetical external variable.
4). Decide to include a hypothetical external variable in the final list of variables that is included in the student's mathematical model.
5). Include a variable in the final list. That will set the method for the value of the source data variable calculating.
At ASR Stage 1 the groups presented in Table 1 were included in the external variables cluster [1].

Table 1. External variables' groups at Stage 1

No.	**Group of variables**	**Planned number of variables in the group**
1	Gender and age	2
2	Geographical position	2–3
3	User interaction parameters and learning style	4–6
4	Academic performance (test results,grades)	1-5
5	Complex skills' level of proficiency	1-5

At Stage 2 of ASR, hypothetical variables of all 5 groups were analyzed.
The progress and results of the analysis of each of the groups are presented further.
The concept of time must be introduced to give an educational trajectory mathematical model a finished form. The trajectory consists of events (states) that follow each other and have two temporal characteristics: interval and duration.

Duration indicates how long a particular state of the trajectory lasted, while the interval reflects how far this state is from the previous one. To unify these two characteristics the period of the absence of any event in the trajectory as a special type event — an inactivity event — is proposed. So, the duration becomes another

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characteristic of the trajectory state. The interval is such a characteristic for the inactivity event.
In accordance with given above the educational trajectory events representation and the hypothesis takes the form of (1, 2):

$$x_t = \{e_t, d_t\} \quad (1)$$
$$h_t = f(ht_{-1}, x_t; \theta) \quad (2)$$

d_t — duration of the condition described by e_t.
To include the characteristics of the continuing time d_t in the mathematical model of the educational trajectory, an event contextualization method was proposed.
In the framework of modeling the characteristics of the duration of the state of the educational trajectory, the vector of the temporary context c^d is calculated first (3):

$$c^d = \varphi (\log (d_t); \theta) \quad (3)$$

φ — nonlinear transformation d_t performed by the direct propagation method in a neural network with a set of parameters θ.
The logarithmic transformation d_t is performed to smooth out a wide range of values expressing time intervals and to make possible the simultaneous use of periods expressed in days, months, and even years [9].
The vector time mask calculation is carried out by linear transformation c^d using a set of weights $W_d \in R^{CxE}$ and offsets $b_d \in \mathbb{R}^E$. The transformation result is then transferred to the nonlinear sigmoidal activation function σ to obtain the mask $m_d \in \mathbb{R}^E$ and $\mathbb{R}^E \rightarrow [0; 1]$ (4). C is the vector dimension of the temporary context, E is the vector dimension of educational trajectory state embedding.

$$m_d = \sigma (c^d W_d + b_d) \quad (4)$$

The temporary mask resulting vector is then superimposed on the educational trajectory state embedding vector by applying the element-wise product operation (5):

$$q_t \leftarrow x_t \,\Theta\, m_d \quad (5)$$

The resulting vector is input to the recursive layer of the RNS. So, the q_t vector simulates one educational path Stage. Q_t vector is used as input for the mathematical model of using the xMANN recurrence network developed in ASR Stage 1 [1]. The educational trajectory model variables change over time, it is dynamic. Using nonlinear transformations in presenting the characteristics of the educational trajectory state duration (4) and the sigmoidal nonlinear activation function, the time mask vector calculation (5) is an educational trajectory non-linear model. During calculating the time mask vector, a set of weights W_d is applied. They are initialized by random variables, which determines the model's stochastic nature. Depending on how the object is represented, the educational trajectory model is functional because of described reasons in relation to the student's model.

At ASR Stage 2, formation the events final list is done. Events are considered by the educational trajectory mathematical model. The events list allows to increase the adequacy of the educational trajectory mathematical model by including in it a wide range of educational activities which are not traditionally included in student models [4-7].

These assessments offer potential educational decisions on the quality of learning outcomes and ensure equity in the learning opportunities' allocation. Investigation of the learning experiences' trajectory supports policy goals, reform and assists the appropriate education system, sets measurable goals. Benchmarking helps countries understand the challenges and risks of student learning, explore their relative weaknesses and strengths, monitor the progress. This helps to correlate learning goals and outcomes, in addition to the proposed

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innovative approach that reflects students' abilities and considers how these skills relate to adulthood [2].

Giving the definition to the learning path, van den Hevel-Panhuizen mention such intertwined meanings:
• overview of the student learning process trajectory;
• a learning trajectory consisting of didactic features that describe how it can best integrate and encourage the learning process and syllabus, indicating which of the major elements of the mathematics curriculum should be taught [26].

Van den Heuvel-Panhuizen describes "learning-to-learn" (TAL8) trajectory [14, 17]:
• it has transitional goals in primary schools;
• it is based on early numerical experiences for children.
• it represents TAL three levels of numbers for the youngest children:
  1)."Emergency number", pre-school year,
  2)."Increasing sense of number", kindergarten 1 and 2,
  3). "Calculations up to 20", 1st and 2nd grade

"Learning-to-learn" trajectory goes beyond the tests and the textbook and it focused on achievement of goals [15, 18, 19]. It is useful in level raising, directing children towards the final basic goals of basic mathematics education. And curricula offer a way to control development of children [3].

AI projections on the path of student learning

Artificial intelligence at schools enables students to analyze performance and abilities, and point ahead with predictive analytics [8]. Developers are developing prediction models of machine learning to achieve this goal: Classification Model, Clustering Model, Forecast Model, Outliers Model, Time Series Model. The programs predict how student will behave in the future if the student letter scores are declining, and he or she does not take appropriate action. Thus, teacher looks for other ways of directing, adapting, or helping students [13, 16, 20].

Table 2. Qualities of learning trajectories and their mis-perceptions [10]

Learning Trajectories Are	**Learning Trajectories Are Not**
Expected probabilities	Stage theories
Domain-specific models	General or universal principles
Empirically-based models of student thinking	Logico-mathematical deconstructions
Connected to big ideas	Curriculum material
Elicited by rich or novel tasks	Derived from typical exercises
Include strategies, reasons, explanations and cases	Sub-goals of the target
A means to avoid errors	Include exploring misconceptions
Ordered by increasing sophistication	Ordered by difficulty
Based in students' thinking over the long term	Based in opinions of experts in mathematics
Evolving	Fixed

Levels in learning trajectories differ qualitatively and the diversity of epistemological objects comprises learning trajectories. Schauble and Lehrer wrote: "Learning progressions are a way to restructure and rethink the content and/or the sequence of the subject matter that is taught. They often serve as proposals to shift our view of what it means to understand an idea..." [10].

V. CONCLUSION

At ASR Stage 2, a mathematical model of the educational trajectory is developed, which includes internal variables that reflect the basic properties of the Stage of the educational trajectory at the time of each event in it; external variables that reflect the student's properties, including student learning outcomes; temporal characteristics of the duration and intervals of Stages that make up the educational trajectory, and thus reflecting their structure.

Based on the work results, student's mathematical model is subject of the educational trajectory and formed including 43 variables reflecting basic students' properties: gender and age variables; 25 variables of parameters of user interaction with the user interface; 4 variables of his training style; 2 variables of complex

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skills' development of both teacher and student; 3 variables of geographical location, including educational facilities and cultural accessibility, and indirect reflection of climatic conditions; 6 variables of academic results, weighting average grades in subjects of profiles and test results considering the time taken to complete the assignment.

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