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Dual Stage Trade Credit Policy for Integrated Inventory System

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Abstract: The present study considers an integrated production inventory system of a supplier and a retailer. The proposed model explores the vendor managed strategy with the dual stage trade credit also known as progressive payment scheme, lead time and shortages. A nonlinear problem is formulated for the purpose of total profit maximization and solved using conventional global optimization method for nonlinear problem. This article can provide the managerial outline for schematizing the production strategy according to the received orders in a joint business enterprise.

Keywords: production, progressive payment, lead time, shortages.

INTRODUCTION

Nowadays extremely competitive market induces the supplier to confront the urge of reducing production cycle time, delivery lead-time and inventory costs. However, each corporation has its individual goal and its specific way of decision-making practice. Owing to the variance among the purposes of each organization and independent decision-making procedures, there has been a requirement of a novel mechanisms to resolve the issues of variation and integrate processes. Integrated supplier- retailer system is one of these mechanisms in which customer's demand is satisfied only through the retailer and each side aims to minimize the ordering, holding and shortage costs over the planning horizon. In such a supply chain, the supplier prefers the retailer to consign orders in advance of his/her need.

However, the retailer wants the supplier to fulfill orders instantaneously devoid of any backlog. Hence, the supply chain deals with a coordination problem in which both the supplier and the retailer want the other contributor to endure the outcome. How should this supply chain's inventory be controlled? Since, the possibility for upgrading within the organization is decreasing; the researchers and leaders of industry are investigating for newer options of integrating the trade operations beyond the organization's boundary. This persuades the supplier to re-evaluate his/her scheme and adopt the policies accordingly. Hence, as an alternative of immediate replenishment, supplier undertakes a lead-time, starts manufacturing after receiving the order and ascertains the delivery of each order in time following the assured lead time. Supplier's assured lead time decreases the retailer's menace from uncertain supply but lengthens the retailer's planning horizon outside the normal replenishment period. Pre-decided orders reduce the supplier's risk from uncertain demand and additional costs. Lead time offers the option to retailer to decide about the policies to avoid having the excess inventory. A cost benefit investigation of this interaction and the consequential inventory costs determine about the lead-time settlement. In order to guarantee the retailer's synchronization to lead-time policy, the supplier may propose the retailer a delay period, known as trade credit period, to settle the account within the predetermined payment period for purchasing cost. This concept provides great incentives to the retailer following the fact that he/she does not require to pay instantly for the purchased items and can hold-up the payment until the end of the permitted time. In case of two level trade credits, the retailer pays no interest during the first credit period. If the payment is deferred beyond the first period, interest would be levied on the left over amount till the next credit period. If the retailer delays the payment further, then extra interest is charged on the remaining amount at the end of second credit offer. The retailer can collect the revenue by selling items and earning interest on the revenue all through the trade credit period and settle the account at the end of credit period. Therefore, the permissibility of delayed payment by the supplier is a special price discount as it causes the reduction in purchase cost and persuades the retailer to elevate the order quantity.

Liao and Shyu [10] supposed that lead time could be reduced through crashing cost. Treville et al. [21] studied the accountability of lead time reduction in progress of demand chain performance. Hsu et al. [7] studied an inventory model with expiration date and uncertain lead time. Leng and Parlar [9] examined lead time reduction in a two-echelon supply chain and compared non-cooperative equilibria vs. coordination under profit-sharing contract. Singh and Singh [18] investigated a supply chain model with stochastic lead time and indefinite partially backlogging and fuzzy ramp-type demand for expiring items. Goyal [6] introduced the permissible delay in payments in the economic ordering quantity model. Davis and Gaither [5] established an economic ordering quantity model where supplier proposes one time opportunity to delay the payments of orders in case an order for additional units is placed. Shah et al. [15] developed model with shortages by extending Goyal's [6] model. Jamal et al. [8] derived an optimal ordering policy with permissible delay in payment and shortages for deteriorating items. Chang and Dye [2] worked on a similar problem with partial backlogging for deteriorating items. Shah et al. [14] explored the impact of permissible delay in payments on economic ordering quantity model assuming stock dependent demand. Singh et al. [19] developed an inventory model for perishable items having quadratic demand, partial backlogging and permissible delay in payments. Gani and Maheswari [11] studied a supply chain model under two levels of trade credit for the retailer's optimal ordering policy in a fuzzy environment. Singh and Singh [17] investigated an integrated supply chain model considering trade credit period for perishable items under imprecise environment. Seifert et al. [13] organized a relevant literature review on permissible delay in payments in Inventory Modelling in detail. Sarker et al. [12] considered supplier's and retailer's trade-credit policy for fixed lifetime products and worked on the inventory model with time varying deterioration. Tyagi [22] worked an EOQ model for flexible trade credit policy for vendor-buyer system. He established three theorems for minimizing the total cost of the system. Tayal et al. [20] worked on an integrated production-distribution model for deteriorating items in a two-echelon supply chain with trade credit. They developed exact cost functions for the vendor and buyer for minimizing the total system cost. Shaikh [16] used the concept of alternative trade credit with variable demand for two warehouses. He used GRG (generalized reduced gradient method) for constrained linear mixed integer problem. Aliabadi et al. [1] implemented con- strained Signomial Geometric Programming (SGP) for integrated inventory system under trade credit. Chung et al. [4] established the inventory ordering quantity model by revisiting the model of Chang and Teng [3] for deteriorating products under conditions including cash discount and progressive payment. Zou and Tian [23] investigated an EOQ model for a supply chain assuming flexible trade credit contract and two-level trade credit policy.

In most of the existing studies, the researchers idealistically supposed that the lead time of the supplier is negligible. Also, the supplier's problem of potential lead time with stock out scenario under progressive payment scheme has been almost ignored. This research is an endeavor to prevail some of the restrictions of the conventional models. In this study, the optimal lead time and the corresponding payment scheme are determined so as to minimize the expected inventory cost offered by the supplier while ensuring the retailer's contribution.

The remaining paper is organized as follows: Section II lists the assumptions and notations. Section III provides the design of the system studied and formulation of the model. In section IV, a numerical example is provided to illustrate the real problem. In final section V, the paper is summarized and concluded.

ASSUMPTIONS AND NOTATIONS

Assumptions

1. Single manufacturer and single buyer are considered.
2. Production rate is a decision variable.
3. The demand rate is exponentially dependent on time.
4. Lead time for the supplier is considered a decision variable.
5. Shortage is allowed at buyer's side.
6. The deterioration rate for each item is constant.
7. The unit production cost is depending on production rate.

Notations

P production rate (independent variable)

$D(t)$ demand rate for the retailer given by ae^{-bt} where a > 0 and if b >0, then rate becomes exponentially decreasing function of time while the nature is reversed when b <0.

Q the retailer's ordering quantity

θ deterioration rate of the item, a constant

L lead time of the supplier (independent variable)

δ the fraction of backordered demand, $0 < \beta < 1$

M_1 the first period of permissible delay in settling account without extra charges

M_2 the second period of permissible delay in settling account with an interest charge, $M_2 > M_1$

I_1 the interest charged per \$ per year by the supplier when the retailer pays during $[M_1, M_2]$

I_2 the interest charged per \$ per year by the supplier when the retailer pays during $[M_1, M_2]$

I_e the interest earned per \$ per year

s the retailer's selling price per unit

p the retailer's purchasing cost per unit

c_{ss} set up cost for the supplier

c unit production cost per unit
c_{hs} holding cost per unit per unit time for the supplier
c_{ds} deterioration cost per unit for the supplier
c_{or} ordering cost for the retailer
c_{hr} holding cost per unit per unit time for the retailer
c_{dr} deterioration cost per unit for the retailer
c_s shortage cost per unit backordered for the retailer
c_l lost sale cost per unit for the retailer
$I_s(t)$ inventory level of the supplier at any time t
$I_r(t)$ inventory level of the retailer at any time t
T the replenishment cycle time

MODEL DEVELOPMENT

Total profit incured on the supplier

Fig. 1 depicts the behavior of change in inventory level at any time 't' for the manufacturer. According to the assumption, the manufacturer starts production at the rate P immediately after receiving the order from the retailer and completes the order by manufacturing the required Q units at time $t = L$. The equation describing the inventory level during production process is $I_s'(t) = P - \theta I_s(t), \qquad 0 \le t \le L$ (1)

Using the initial condition $I_s(0) = 0$, the solution of above equation is

$$I_s(t) = \frac{P}{\theta}\left(1 - e^{-\theta t}\right), \qquad 0 \le t \le L \tag{2}$$

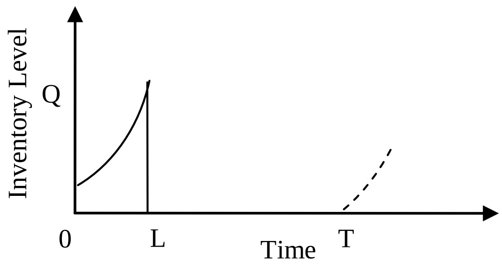

Fig.1: Supplier's inventory system

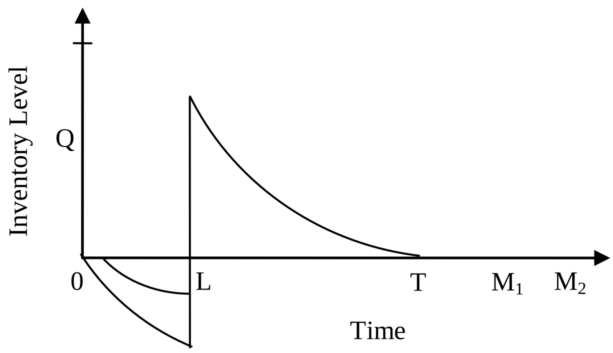

Fig.2: Retaliler's inventory system

Also, the boundary condition $I_s(L) = Q$ implies $Q = \frac{P}{\theta}\left(1 - e^{-\theta L}\right)$ (3)

Holding cost of the supplier is $HC_s = c_{hs}\int_0^L I_s(t)\,dt = \frac{c_{hs}P}{\theta}\left\{L + \frac{1}{\theta}\left(e^{-\theta L} - 1\right)\right\}$ (4)

Deterioration cost of the supplier is

$$DC_s = c_{ds}\int_0^L \theta I_s(t)\,dt = c_{ds}P\left\{L + \frac{1}{\theta}\left(e^{-\theta L} - 1\right)\right\} \tag{5}$$

Production cost of the supplier is

$$PC_s = cPL \tag{6}$$

Revenue of the supplier is pQ.

The total expected profit of the supplier including set up cost, holding cost and production cost is given by

$$TP_s = \frac{1}{T}\left[pQ - c_{ss} - \left(\frac{c_{hs}}{\theta} + c_{ds} \right) P\left\{ L + \frac{1}{\theta}\left(e^{-\theta L} - 1 \right) \right\} - cPL \right] \tag{7}$$

Total profit incurred on the retailer

The retailer placed an order at the time $t = 0$. Due to the supplier's lead time, the lot size is delivered to the retailer at the time $t = L$. The shortage accumulates during this time which is partially backordered. After receiving the order, the retailer fulfils the backlogged demand initially. Then inventory depletes due to the joint effect of demand and deterioration and becomes zero at time $t = T$ (See Fig. 2).

Therefore, the change in inventory level of the retailer at any time 't' can be expressed as:

$$I_{r1}'(t) = -\delta a e^{-bt}, \quad 0 \le t \le L \tag{8}$$

$$I_{r2}'(t) = -\theta I_{r2}(t) - a e^{-bt}, \quad L \le t < T \tag{9}$$

Using the initial condition $I_{r1}(0) = 0$, the solution of eq. (8) is

$$I_{r1}(t) = -\frac{\delta a}{b}\left(1 - e^{-bt}\right), \quad 0 \le t \le L \tag{10}$$

The total backordered demand stock at the time $t = L$, is

$$B = -I_{r1}(L) = \frac{\delta a}{b}\left(1 - e^{-bL}\right) \tag{11}$$

The stock left at the time $t = L$, after fulfilling the backordered demand is $q = Q - B = Q - \frac{\delta a}{b}\left(1 - e^{-bL}\right)$
(12)

Using the initial condition $I_{r2}(L) = q$, the solution of eq. (9) is

$$I_{r2}(t) = \left\{ q + \frac{a e^{-bL}}{(\theta - b)} \right\} e^{-\theta(t-L)} - \frac{a e^{-bt}}{(\theta - b)}, \quad L \le t \le T \tag{13}$$

Also, the boundary condition $I_{r2}(T) = 0$ implies

$$q = \frac{a}{(\theta - b)}\left(e^{-\theta L} e^{(\theta - b)T} - e^{-bL} \right) \tag{14}$$

The total ordered quantity is given by

$$Q = \frac{a}{(\theta - b)}\left(e^{-\theta L} e^{(\theta - b)T} - e^{-bL} \right) + \frac{\delta a}{b}\left(1 - e^{-bL}\right) \tag{15}$$

Holding cost of the retailer is

$$HC_r = c_{hr}\int_L^T I_{r2}(t)\,dt = c_{hr}\left[-\frac{1}{\theta}\left(q + \frac{a e^{-bL}}{\theta - b} \right)\left(e^{-\theta(T-L)} - 1 \right) + \frac{a}{b(\theta - b)}\left(e^{-bT} - e^{-bL} \right) \right] \tag{16}$$

Deterioration cost of the retailer is

$$DC_r = c_{dr}\int_L^T \theta I_{r2}(t)\,dt = c_{dr}\theta\left[-\frac{1}{\theta}\left(q + \frac{a e^{-bL}}{\theta - b} \right)\left(e^{-\theta(T-L)} - 1 \right) + \frac{a}{b(\theta - b)}\left(e^{-bT} - e^{-bL} \right) \right] \tag{17}$$

Shortage cost for backordered demand of the retailer

$$SC_r = c_s\int_0^L \left(-I_{r1}(t) \right) dt = \frac{c_s \delta a}{b}\left\{ L + \frac{1}{b}\left(e^{-bL} - 1 \right) \right\} \tag{18}$$

Lost sale cost of the retailer is

$$LC_r = c_l\int_0^L (1 - \delta) a e^{-bt}\,dt = -c_l(1 - \delta)\frac{a}{b}\left(e^{-bL} - 1 \right) \tag{19}$$

The ordering cost of the retailer is $OC_r = c_{or}$.

The total units sold by the retailer till $t=T$ is given by $B + \int_L^T a e^{-bt}\,dt = B - \frac{a}{b}\left(e^{-bT} - e^{-bL} \right)$. The retailer's revenue is

$$SR_r = s\left\{ B - \frac{a}{b}\left(e^{-bT} - e^{-bL} \right) \right\} \tag{20}$$

The retailer's purchasing cost is pQ. Regarding interests charged and earned, based on the length of the replenishment cycle T, we have three possible cases: (1) $T \le M_1$, (2) $M_1 \le T \le M_2$, and (3) $T \ge M_2$.

a) Case 1. $T \le M_1$

Since, the retailer has to pay the amount equals to pQ till $t = M_1$. Hence, the interest charged by the supplier denoted by IC_1 is zero. However, the retailer deposits the total revenue into an account and earns interest at the rate I_e per unit \$ per year. The total interest earned denoted by IE_1 is given by

$$IE_1 = sI_e\left[\int_L^T\left(B+ae^{-bt}\right)tdt + \left\{B-\frac{a}{b}\left(e^{-bT}-e^{-bL}\right)\right\}\left(M_1-T\right)\right]$$

$$= sI_e\begin{bmatrix}-\frac{a}{b}\left\{Te^{-bT}-Le^{-bL}+\frac{1}{b}\left(e^{-bT}-e^{-bL}\right)\right\} \\ +\frac{B}{2}\left(T^2-L^2\right)+\left\{B-\frac{a}{b}\left(e^{-bT}-e^{-bL}\right)\right\}\left(M_1-T\right)\end{bmatrix} \tag{21}$$

The total expected profit per unit time of the retailer is

$$TP_{r1} = \frac{1}{T}\left(SR_r + IE_1 - OC_r - HC_r - DC_r - SC_r - LC_r - IC_1\right) \tag{22}$$

The total expected profit per unit time of the supplier is

$$TP_{s1} = \frac{1}{T}\left[pQ - c_{ss} - \left(\frac{c_{hs}}{\theta}+c_{ds}\right)P\left\{L+\frac{1}{\theta}\left(e^{-\theta L}-1\right)\right\} - cPL + IC_1\right] \tag{23}$$

The total integrated profit of the system is $TP_1 = TP_{r1} + TP_{s1}$ (24)

b) Case 2 $M_1 \le T \le M_2$

During the time interval $\left[L,\ M_1\right]$, the retailer sells the products and deposits the revenue into an account that earns interest at the rate I_e per unit \$ per year. The total interest earned during this period is

$$IE_2 = sI_e\int_L^{M_1}\left(B+ae^{-bt}\right)tdt = sI_e\left[-\frac{a}{b}\left\{M_1e^{-bM_1}-Le^{-bL}+\frac{1}{b}\left(e^{-bM_1}-e^{-bL}\right)\right\}+\frac{B}{2}\left(M_1^{\,2}-L^2\right)\right] \tag{25}$$

The retailer sells $B-\frac{a}{b}\left(e^{-bM_1}-e^{-bL}\right)$ units till $t = M_1$. Therefore, he has the amount $s\left\{B-\frac{a}{b}\left(e^{-bM_1}-e^{-bL}\right)\right\}$, plus the interest earned given by eq. (21) in the account. Accordingly, there are two possible sub-cases:

1) Sub-case 2.1

$$s\left\{B-\frac{a}{b}\left(e^{-bM_1}-e^{-bL}\right)\right\}+sI_e\left[-\frac{a}{b}\left\{M_1e^{-bM_1}-Le^{-bL}+\frac{1}{b}\left(e^{-bM_1}-e^{-bL}\right)\right\}+\frac{B}{2}\left(M_1^{\,2}-L^2\right)\right] \ge pQ$$

In this sub-case, the retailer has the money in his account to pay off the total purchase cost at time M_1. Hence, the total purchase cost will be paid at M_1 and the payable interest denoted by $IC_{2.1}$ is zero. The interest earned denoted by,

$$IE_{2.1} = IE_2 \tag{26}$$

The total expected profit of the retailer is

$$TP_{r2.1} = \frac{1}{T}\left(SR_r + IE_{2.1} - HC_r - DC_r - SC_r - LC_r - PC_r - OC_r - IC_{2.1}\right) \tag{27}$$

The total expected profit of the supplier is $TP_{s2.1} = \frac{1}{T}\left[pQ - c_{ss} - \left(\frac{c_{hs}}{\theta}+c_{ds}\right)P\left\{L+\frac{1}{\theta}\left(e^{-\theta L}-1\right)\right\} - cPL + IC_{2.1}\right]$ (28)

The total integrated profit of the system is $TP_{2.1} = TP_{r2.1} + TP_{s2.1}$ (29)

2) Sub-case 2.2

$$s\left\{B-\frac{a}{b}\left(e^{-bM_1}-e^{-bL}\right)\right\}+sI_e\left[-\frac{a}{b}\left\{M_1e^{-bM_1}-Le^{-bL}+\frac{1}{b}\left(e^{-bM_1}-e^{-bL}\right)\right\}+\frac{B}{2}\left(M_1^{\,2}-L^2\right)\right] < pQ$$

In this sub-case, the retailer has not enough money in the account to pay off the total purchase cost at time M_1. However, the remaining amount can be paid till the time $t = M_2$ with the interest charged at the rate I_1 per dollar per year on the unpaid balance denoted by L_1 where

$$L_1 = pQ - s\left\{B - \frac{a}{b}\left(e^{-bM_1} - e^{-bL}\right)\right\} - sI_e\left[-\frac{a}{b}\left\{M_1 e^{-bM_1} - Le^{-bL} + \frac{1}{b}\left(e^{-bM_1} - e^{-bL}\right)\right\} + \frac{B}{2}\left(M_1^{\ 2} - L^2\right)\right] \quad (30)$$

The interest payable is $IC_{2.2} = \frac{L_1 I_1}{pQ}\int_{M_1}^{T} I_{r1}(t)\,dt$ (31)

The interest earned, $IE_{2.2} = IE_2$ (32)

The total expected profit of the retailer is $TP_{r2.2} = \frac{1}{T}\left(SR_r + IE_2 - HC_r - DC_r - SC_r - LC_r - PC_r - OC_r - IC_{2.2}\right)$ (33)

The total expected profit of the supplier is

$$TP_{s2.2} = \frac{1}{T}\left[\begin{array}{l} pQ - c_{ss} - \left(\frac{c_{hs}}{\theta} + c_{ds}\right)P\left\{L + \frac{1}{\theta}\left(e^{-\theta L} - 1\right)\right\} \\ -cPL + IC_{2.2} \end{array}\right] \quad (34)$$

The total integrated profit of the system is $TP_{2.2} = TP_{r2.2} + TP_{s2.2}$ (35)

c) Case 3 $T \geq M_2$

Let R_1 be the total amount of money in the account of the retailer at $t = M_1$, then

$$R_1 = s\left\{B - \frac{a}{b}\left(e^{-bM_1} - e^{-bL}\right)\right\} + sI_e\left[-\frac{a}{b}\left\{M_1 e^{-bM_1} - Le^{-bL} + \frac{1}{b}\left(e^{-bM_1} - e^{-bL}\right)\right\} + \frac{B}{2}\left(M_1^{\ 2} - L^2\right)\right] \quad (36)$$

There are three possible sub-cases:

1) Sub-case 3.1. $R_1 \geq pQ$

This sub-case is similar to sub-case 2.1. the retailer has the money in his account to pay off the total purchase cost at time $t = M_1$. Therefore, $TP_{r3.1} = TP_{r2.1}$ and $TP_{s3.1} = TP_{s2.1}$

The total integrated profit is same as given by eq. (29).

If $R_1 < pQ$, then the retailer has not enough money in his account to pay off the total purchase cost at time $t = M_1$ but he can pay the total purchase cost before/on or after $t = M_2$. Hence, the retailer pays the amount R_1 at M_1 and the supplier starts charging the interest at the rate I_1 per dollar per year on the unpaid balance is given by the amount L_1. Let R_2 be the revenue of the retailer during the time interval $\left[M_1,\ M_2\right]$,

then $R_2 = -\frac{sa}{b}\left(e^{-bM_2} - e^{-bM_1}\right)$ (37)

2) Sub-case 3.2. $R_1 < pQ$ and $R_2 \geq pQ - R_1$

In this sub-case, the retailer can pay the unpaid balance L_1 before or on $t = M_2$ with the interest payable per year, $IC_{3.2} = \frac{L_1 I_1}{TpQ}\int_{M_2}^{M_1} I_{r1}(t)\,dt$ (38)

The total interest earned per year $IE_{3.2} = \frac{IE_2}{T}$ (39)

The total expected profit of the retailer is $TP_{r3.2} = SR_r + IE_{3.2} - HC_r - DC_r - SC_r - LC_r - PC_r - OC_r - IC_{3.2}$ (40)

The total expected profit of the supplier is

$$TP_{s3.2} = pQ - c_{ss} - \left(\frac{c_{hs}}{\theta} + c_{ds}\right)P\left[L + \frac{1}{\theta}\left(e^{-\theta L} - 1\right)\right] - cPL + IC_{3.2} \quad (41)$$

The total integrated profit of the system is

$$TP_{3.2} = TP_{r3.2} + TP_{s3.2} \quad (42)$$

3) Sub-case 3.3 $R_1 < pQ$ and $R_2 < pQ - R_1$

In this sub-case, the retailer has not enough money in his account to pay off the total purchase cost at time $t = M_2$. The retailer pays the amount R_1 at M_1 and the amount R_2 at M_2. The interest is charged at the rate I_1 per dollar per year by the supplier on the unpaid balance L_1 during the time interval $\left[M_1,\ M_2\right]$. Further, the interest is charged at the rate I_2 per dollar per year by the supplier on the remaining unpaid amount L_2 (say) till the time $t = T$ where $L_2 = pQ - R_1 - R_2$. The total interest charged per year denoted by $IC_{3.3}$ is given by

$$IC_{3.3} = \frac{I_1 L_1}{T}\left(M_2 - M_1\right) + \frac{I_2 L_2}{T}\left(T - M_2\right) \tag{43}$$

The total interest earned per year $IE_{3.3} = \frac{IE_2}{T}$ (44)

The total expected profit of the retailer is $TP_{r3.3} = SR_r + IE_3 - HC_r - DC_r - SC_r - LC_r - PC_r - OC_r - IC_{3.3}$ (45)

The total expected profit of the supplier is

$$TP_{s3.3} = pQ - c_{ss} - \left(\frac{c_{hs}}{\theta} + c_{ds}\right) P\left[L + \frac{1}{\theta}\left(e^{-\theta L} - 1\right)\right] - cPL + IC_{3.3} \tag{46}$$

The total integrated profit of the system is

$$TP_{3.3} = TP_{r3.3} + TP_{s3.3} \tag{47}$$

The problem is to maximize the total integrated profit in each case subjected to the non-negative restriction of the decision variables.

Numerical Illustration and Sensitivity Analysis

To illustrate the proposed model, the numerical experiments are performed for each case with the exponential demand using the Nelder Mead Algorithm to find the global optimal solution with the help of Mathematica 12 and the following values of different parameters described in section II are used in appropriate units:

a=50, b= -0.02, θ= 0.07, δ=0.6, M_1= 30 days (30/365 years), M_2= 40days (90/365 years), I_1= 0.14, I_2= 0.17, I_e= 0.12, s=40, p= 20, c_{ss}= 80, c_{hs}= 0.5, c_{ds}= 0.6, c_{or}= 100, c_{hr}= 0.5, c_{dr}= 0.6, c_s= 1, c_l= 1

The above data is used in each case and the corresponding results are summarized as below:

Table 1: Results obtained in different cases

Case	L	T	TP
1	38009.6871	16.9851	29.9999
2.1	38009.6871	16.9851	30.0001
2.2	38009.6871	16.9851	30.0001
3.1	31758.8441	16.0747	40.0001
3.2	31758.8441	16.0747	40.0001
3.3	31758.8441	16.0747	40.0001

Further, analysis of the result corresponding to decreasing demand and the maximum profit can be obtained in case 1 & 2.

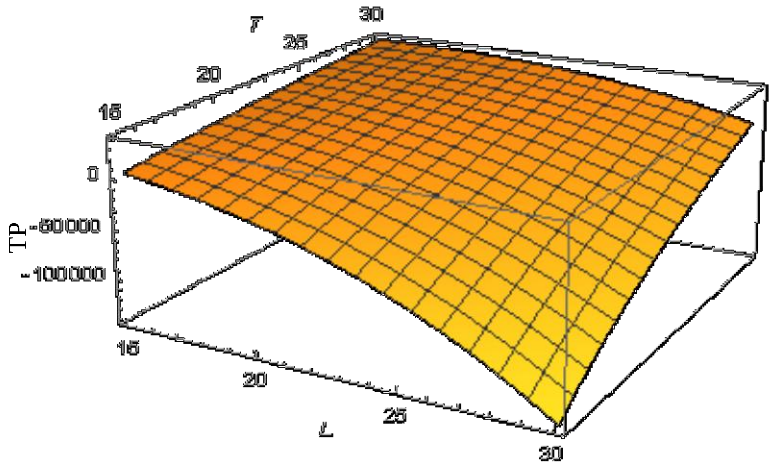

Fig.3: Convexity of TP w.r.t. L and T for case 1

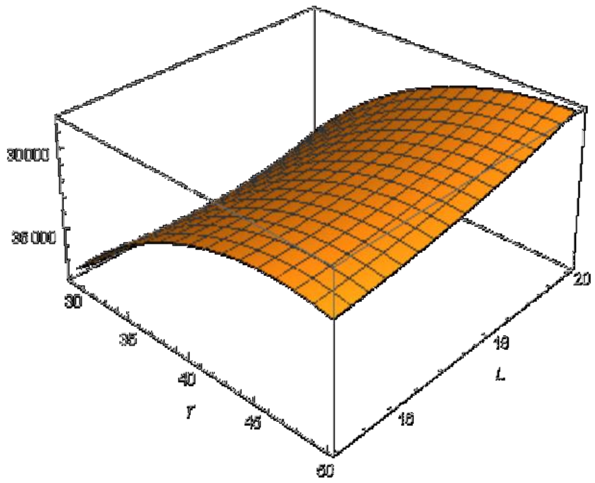

Fig.4: Convexity of TP w.r.t. L and T for case 2.1

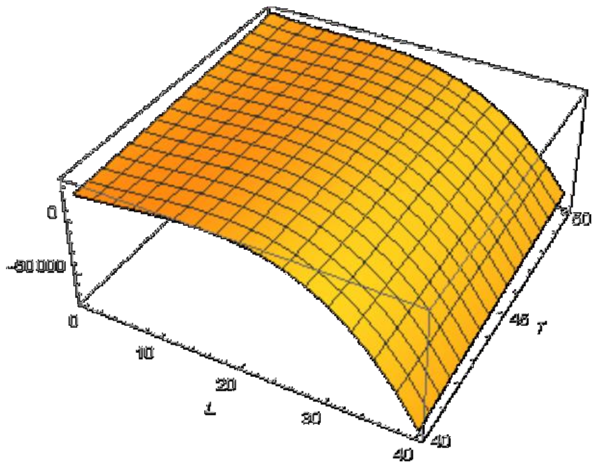

Fig.5: Convexity of TP w.r.t. L and T for case 3.1

Sensitivity analysis

Sensitivity analysis is performed corresponding to this particular value by shifting the parameters by 50% and 25% taking one parameter at a time and keeping all other parameters unchanged. Impact of various parameters on replenishment policy is examined and summarized in table 2-5.

Table 2: Effect of credit period 'M1'

M_1	*L*	*T*	*TP*
-50%	13.769484757843466	13.769484757843466	27290.74840786529
-25%	21.892838572287886	37.49999999899746	58868.91028844737
+25%	39.16511301301301	62.49999999899572	180456.99926683362
+50%	7.996110999014303`*^7	74.99999997248108	205511.58085913633

Table 3: Effect of demand sensitive parameter 'a'

a	*L*	*T*	*TP*
-50%	16.985149276075948	29.999999998991687	19001.843598337302
-25%	16.985149276075948	29.999999998991687	28505.765397506064
+25%	16.985149276075944	29.999999998991683	38045.79925494261
+50%	16.985149276075944	29.999999998991683	47513.60899584354

Table 4: Effect of deterioration rate 'θ'

θ	L	T	TP
-50%	17.108321885590325	29.9999999899848	38899.44581931822
-25%	17.061676839748102	29.999999998999336	38269.46494048082
+25%	16.877751063843313	29.999999998999183	37909.7420368312
+50%	16.731667938575946	29.999999999839275	37903.91087019982

Table 5: Effect of the coefficient of backordered demand 'δ'

δ	L	T	TP
-50%	15.272034289356943	29.999999999	22882.24716938028
-25%	16.44296717048156	29.999999998949484	30381.109376609707
+25%	17.299646492579864	29.9999999899904	45687.52218264019
+50%	17.505332039703728	29.999999998992525	53389.40786861061

- Table 2 shows that the increase in credit periods results into increase in total profit of the integrated system. It is quite justified because increasing credit period induces the retailer to buy more quantity which in turn induces more profit.
- As the first credit period increases, the production rate decreases. During first credit period, no interest is chargeable to the retailer. Therefore, in order to maintain his profit, the supplier manufactures less quantity in more time. Hence, the production rate becomes smaller and lead time becomes larger.
- Table 3 shows the effect of increasing initial parameter 'a' of demand. High demand of manufactured entity stimulates more production which in turn becomes a significant cause to earn extra profit.
- Table 4 shows that higher deterioration rate results into lower total profit as the product's deterioration cost gets added to the total cost of the system.
- Increase in backordered demand gives ample boost to the total profit, this scenario can be clearly understood from table 5 as there would be decrease in goodwill loss and technically profit gets higher due to increased sale.

CONCLUDING REMARKS

In this research we have studied a two echelon supply chain system with some very realistic assumptions from the perspective of a supplier and the retailer. The whole model is studied in an inflationary environment. This assumption certainly adds an extra factor of the real market to our study. Supplier is not always in the position of fulfilling a customer's request at any time, thus existence of lead time brings the study in close proximity to reality. Every supplier wants to minimize his holding cost as well as the deterioration cost, thus force to adopt new policies to deal with the situation. In this article, a situation where in order to reduce his holding cost and the deterioration cost supplier don't hold the inventory indeed but produce the items whenever they are demanded. As a result, retailer is compelled to have the shortage for the period. Since the retailer's shortage takes place due to the supplier's lead time. Therefore, for the sake of convenience, the retailer is offered a progressive credit period to settle the account by the supplier. Since retailer is not restricted to pay immediately for the quantities he ordered, hence he can take his time, sells the items and earns the interest on the revenue caused by selling of the items. It is also observed that if supplier does not offer any credit period and bound the retailer to have the shortages due to his lead time, the retailer suffers from huge loss of money. Therefore, progressive payment scheme with supplier's lead time is a win-win situation for both retailer and the supplier and is good for the prolonged existence of the supply chain.

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